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Intersection statements for systems of sets. (In English)

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A collection of sets is called a Δ-system if any two sets have the same intersection. Let $f(k,r)$ be the least integer such that any collelction of $f(k,r)$ k-element sets contains a Δ-system consisting of r sets. *P. Erdős* and *R. Rado* [J. Lond. Math. Soc. 44, 467- 479 (1969; Zbl 172.29601)] proved that $(r-1)^k < f(k,r) < k!(r-1)^k$ and conjectured that $f(k,r) < C^k$ for some constant C. Erdős offered \$1000 for a proof or disproof of this for $r = 3$.

The paper under review concerns a related problem. Let $F(n,r)$ be the greatest integer such that there exists a collection of subsets of an n-element set which does not contain a Δ-system consiting of r sets. *P. Erdős* and *E. Szemerédi* [J. Comb. Theory, Ser. A 24, 308-313 (1978; Zbl 383.05002)] showed that $F(n,3) < 2^{n-\sqrt{n}/10}$ and $F(n,r) > (1+c_r)^n$, where the constant $c_r \to 1$ as $r \to \infty$. The authors provide new lower bounds for $F(n,r)$ which are constructive and improve the previous best probabilistic results. They also prove a new upper bound. Moreover, for certain n it is shown that $F(n,3) \geq 1.551^{n-2}$.

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