Zbl 842.41003

Erdős, Paul; Szabados, J.; Vértesi, P.

On the integral of the Lebesgue function of interpolation. II. (In English)

Acta Math. Hung. 68, No.1-2, 1-6 (1995). [0236-5294]

Notations. $-1 \le x_{0,n} < x_{1,n} < \cdots < x_{1,n} < 1$ is a set of nodes on the interval $[-1,1]$. For brevity set $x_k := x_{k,n}$. Define also some well known quantities

$$\ell_k(x) := \ell_{k,n}(x) = \frac{\omega(x)}{\omega'(x_k)(x-x_k)}, \qquad \omega(x) = \prod_{k=0}^{n}(x-x_k),$$

$\lambda(a,b) := \max_{a\le x\le b}\sum_{k=1}^{n}|\ell_k(x)|$, $-1 \le a < b \le 1$. The present paper and a former paper by the first two authors [Acta Math. Acad. Sci. Hungar 32, 191-195 (1978; Zbl 391.41003)] deal with lower bound estimates of the function $\lambda_n(x) := \sum_{k=0}^{n}|\ell_k(x)|$. In the above mentioned paper it was shown that for any interval $[a,b] \subseteq [-1,1]$ and arbitrary nodes x_k the inequality

$$\int_a^b \sum_{k=0}^{n}|\ell_k(x)|dx \ge c(b-a)\log n$$

holds for sufficiently large n depending only on the interval $[a,b]$. This inequality was an improvement of Bernstein's $\lambda_n(a,b) \ge c_1 \log n,\ n \ge n_1(a,b)$. A further improvement is shown in the present paper, namely a similar inequality is derived for every individual interval $[a_n,b_n] \subseteq [-1,1]$ and for all n without exception. The result states

Theorem. There exists an absolute positive constant c for which the inequality

$$\int_{a_n}^{b_n} \lambda(x)dx \ge c(b_n-a_n)\log(n(\alpha_n-\beta_n)+2), \qquad (a_n \cos\alpha_n,\ b_n = \cos\beta_n).$$

In fact the authors show the sharpness, in a sense, of their estimate by showing that

$$\max_{a_n\le x\le b_n} \lambda_n(x) = O\big(\log(n(\alpha_n-\beta_n)+2)\big).$$

Z.Rubinstein (Haifa)

Classification:
41A05 Interpolation