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On sum sets of Sidon sets. II. (In English)

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Let $A \subseteq \mathbb{N} = \{1, 2, \dots\}$ and $S_A = \{a + a' \mid a, a' \in A\}$. If for every $n \in \mathbb{N}$ the equation $a + a' = n$; $a \le a'$; $a, a' \in A$ has at most one solution then A is called a Sidon set.

At first blocks of consecutive elements in S_A for Sidon sets A are studied. For $n \in \mathbb{N}$ let $H(n) = \max\{h \in \mathbb{N} \mid \{m+1, m+2, \dots, m+h\} \subseteq S_A,\ m \le n\}$ taken over all Sidon sets $A \subseteq \{1, 2, \dots, n\}$. It is shown that $n^{1/3} \ll H(n) \ll n^{1/2}$. The lower bound is obtained by construction of a suitable infinite Sidon set while the upper bound is a consequence of the following much sharper result, choosing $l = [200n^{1/2}]$: For all Sidon sets $A \subseteq \{1, 2, \dots, n\}$ and all $l \in \mathbb{N}$, $k \in \mathbb{Z}$ we have

$$|S_A \cap [k+1, k+l]| < \tfrac{1}{2} l + 7 l^{1/2} n^{1/4}.$$

Let $n \in \mathbb{N}$. For $A \subseteq \mathbb{N}$, $|A| = n$ the minimum of $|S_A|$ is obtained by arithmetic progressions A and the maximum by Sidon sets A. Therefore one can expect that a well-covering of a Sidon set by arithmetic progressions is impossible, even by generalized arithmetic progressions $P = \{e + x_1 f_1 + \cdots + x_m f_m \mid x_i \in \{1, \dots, l_i\}$ for $i = 1, \dots, m\}$, where $m, l_1, \dots, l_m \in \mathbb{N}$; $e, f_1, \dots, f_m \in \mathbb{Z}$. Let $\dim P = m$ and $Q(P) = l_1 l_2 \dots l_m$ be the dimension and size of P. A measure of well-covering for A by generalized arithmetic progressions (g.a.p.) of dimension m is given by the minimum $D_m(A)$ of the terms $t \sum_{j=1}^{t} Q(P_j)$ taken over all coverings $A \subseteq \bigcup_{j=1}^{t} P_j$ where P_j are g.a.p. with $\dim P_j = m$ for $j = 1, \dots, t$. If $D_m(A)$ is close to $|A|$ then A can be covered by "few" g.a.p.. If $D_m(a)$ is close to $|A|^2$ we have the opposite situation. For all finite Sidon sets A it is shown that $D_m(A) > 2^{-m-1}|A|^2$. On the other hand for all $m \in \mathbb{N}$ there exists a finite Sidon set A such that $D_m(A) \le \frac{1}{2}|A|^2$. These two theorems are proved in a more general form for $B_2[g]$ sets.

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