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Similar configurations and pseudo grids. (In English)

Böröczky, K. (ed.) et al., Intuitive geometry. Proceedings of the 3rd international conference held in Szeged, Hungary, from 2 to 7 September, 1991. Amsterdam: North-Holland, Colloq. Math. Soc. János Bolyai. 63, 85-104 (1994). [ISBN 0-444-81906-1/hbk]

Given a finite set $\mathcal{P}$ of points in the plane, it is of interest to find another, n-point, set $\mathcal{A}$ which contains many similar or homothetic copies of $\mathcal{P}$. When $\mathcal{P}$ is a square or equilateral triangle, it is easily seen that a suitable regular grid achieves $c \cdot n^2$ similar copies and $c \cdot n^{3/2}$ homothetic copies.

Such constructions might appear to depend on the fact that the square and equilateral triangle tile the plane in a lattice tiling. The authors show that this is not the case. If $\mathcal{P}$ is a triangle, or can be represented in the complex plane by a set of algebraic complex numbers $\{\xi_i\}$, then there exist "pseudogrids" containing $c \cdot n^2$ similar copies or $c \cdot n^{3/2}$ homothetic copies. The points of these pseudogrids are the values of polynomials in $\{\xi_i\}$ with certain bounds on coefficients and degree. Even if $\mathcal{P}$ cannot be so represented, $O(n^2)$ similar copies and $O(n^{3/2})$ homothetic copies may be achieved.

The results for homothetic copies are extended to higher dimensions. If R^d it is possible to construct pseudogrids which contain $c \cdot n^{(d+1)/d}$ homothetic copies of a pattern $\mathcal{P}$ with algebraic coordinates, or $O(n^{(d+1)/d})$ homothetic copies of an arbitrary pattern.

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