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Erdős, Paul; Sárközy, A.
Arithmetic progressions in subset sums. (In English)
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For any set $\mathcal{A}$ of positive integers, denote by $\mathcal{P}(\mathcal{A})$ the set of positive integers n which can be expressed as a sum of distinct elements of $\mathcal{A}$. Let $u = F(N,t)$ be the greatest integer u such that for every $\mathcal{A} \subset \{1,2,\dots,N\}$ with $|\mathcal{A}| = t$, the set $\mathcal{P}(\mathcal{A})$ contains u consecutive multiples of a positive integer d: $\{(x+1)d,(x+2)d,\dots,(x+u)d\} \subset \mathcal{P}(\mathcal{A})$, for some x and d, and let $v = G(N,t)$ be the greatest integer v such that for every $\mathcal{A} \subset \{1,2,\dots,N\}$ with $|\mathcal{A}| = t$, the set $\mathcal{P}(\mathcal{A})$ contains an arithmetic progression of length v. It is clear that $F(N,t) \le G(N,t)$ for all N,t.
Extending earlier work of Sárközy, the authors prove:
I. If $N \ge N_0$ and $18(\log N)^2 < t \le N$. Then $F(N,t) > \frac{1}{18}\frac{t}{(\log N)^2}$.
II. (i). If $N > N_0$ and $c\log N < t < \frac{1}{3}N^{1/3}$, then $F(N,t) < 16\frac{t}{\log N}\log(\frac{t}{\log N})$.
(ii). If $\varepsilon > 0$ and $t_0(\varepsilon) < t < (1-\varepsilon)N^{1/2}$, then $F(N,t) < (1+\varepsilon)t$.
III. (i). If $N > N_0$ and $\exp(2(\log N)^{1/2}) < t < N^{1/2}$, then

$$G(N,t) < t\exp\left(4\max\left(\frac{\log N}{\log t},\frac{(\log t)^2}{\log N}\right)\right).$$

(ii). For all $t_0 < t < \frac{1}{2}N^{1/2}$, $G(N,t) < 2t^{3/2}$.
Finally, upper and lower bounds are also obtained for certain other functions associated with 3 term arithmetic progressions.

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11B25 Arithmetic progressions

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