Zbl 663.05013
Erdős, Paul
Some old and new problems in combinatorial geometry. (In English)
Applications of discrete mathematics, Proc. 3rd SIAM Conf., Clemson/South Carolina 1986, 32-44 (1988).

[For the entire collection see Zbl 655.00007.]
Let $x_1, x_2, ..., x_n$ be n distinct points in a metric space. Usually we will restrict ourselves to the plane. Denote by $D(x_1, ..., x_n)$ the number of distinct distances determined by $x_1, ..., x_n$. Assume that the points are in r-dimensional space. Denote by

$$f_r(n) = \min_{x_1,...,x_n} D(x_1, .., x_n).$$

I conjectured more than 40 years ago that $f_2(n) > c_1 n(\log n)^{1/2}$. The lattice points show that this if true is best possible. In this paper we discuss problems related to the conjecture and other questions related to this parameter.

Classification:
05B25 Finite geometries (combinatorics)
05-02 Research monographs (combinatorics)
00A07 Problem books

Keywords:
distances; lattice points