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Erdős, Paul; Révész, P.
On the area of the circles covered by a random walk. (In English)
J. Multivariate Anal. 27, No.1, 169-180 (1988). [0047-259X]

A simply symmetric random walk on the plane is considered. Let

$$Q(N) = \{x = (i, j): \quad ||x|| = (i^2 + j^2)^{1/2} \leq N\}.$$

The circle $Q(N)$ is covered by the random walk in time n if $\xi(x, n) > 0$ for every $x \in Q(N)$ where $\xi(x, n)$ means the number of passings through the point x during time n. Let $R(n)$ be the largest integer for which $Q(R(n))$ is covered in n. For $R(n)$ the following lower estimate is proved:
for any $\epsilon > 0$ $R(n) \geq \exp((\log n)^{1/2}/(\log_2 n)^{3/4+\epsilon})$ a.s. for all finitely many n where $\log_k$ is the k times iterated logarithm. An estimate is obtained for the density $K(N, n)$ of the points of $Q(N)$ covered by the random walk. Some further related problems are formulated.

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60J15 Random walk
60F15 Strong limit theorems
60G17 Sample path properties

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random walk on the plane; iterated logarithm