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On nilpotent but not abelian groups and abelian but not cyclic groups. (In English)

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Using general sieve-type methods of number theory and certain density estimates for prime numbers, the authors derive asymptotic formulae for $A(n) - C(n)$ and $N(n) - A(n)$, where $A(n) = \#\{m \leq n$: every group of order m is abelian$\}$, $C(n) = \#\{m \leq n$: every group of order m is cyclic$\}$, and $N(n) = \#\{m \leq n$: every group of order m is nilpotent$\}$.

The second author [Arch. Math. 31, 536-538 (1978; Zbl 388.20021)] and *E. J. Scourfield* [Acta Arith. 29, 401-423 (1976; Zbl 286.10023)] showed previously that asymptotically all three of the above counting functions have the form $(1 + o(1))ne^{-\gamma}/\log_3 n$.

The present authors now prove that there exist constants c_1, c_2 such that

$$A(n) - C(n) = (1 + o(1))c_1 n/(\log_2 n)(\log_3 n)^2,$$

$$N(n) - A(n) = (1 + o(1))c_2 n/(\log_2 n)^2(\log_3 n)^2.$$

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Classification:
11N45 Asymptotic results on counting functions for other structures
20K99 Abelian groups
20D99 Abstract finite groups

Keywords:
asymptotic formulae