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**Erdős, Paul; Odlyzko, Andrew M.; Sárközy, A.**
*On the residues of products of prime numbers.* (In English)
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This paper contains a modest attack to the problem proposed by *P.Erdős* that for any sufficiently large prime $q$ and any residue class $a \not\equiv 0$ modulo $q$ the congruence $p_1p_2 \equiv a(\text{mod} q)$ can be solved in primes $p_1 \leq q$ and $p_2 \leq q$. All considerations are subject to the quasi-Riemann hypothesis $H(\theta_q, x)$, i.e., it is supposed that for all characters $\chi$ modulo $q$ the $L(s,\chi)$ do not vanish in the domain Re $s > \theta_q$, $|\text{Im } s| < x^{1-\theta_q}$.
The generalized Riemann hypothesis is $H(1/2,\infty)$ but this is not enough to imply the above conjecture. There are three possible ways to weaken it, which can be satisfied (i) with almost all residue classes mod $q$, (ii) with the product of three primes instead of two, and (iii) with a little bit larger primes $p_1$ and $p_2$.
It is proved that
(i) if $H(\theta_q, q)$ is true then $p_1p_2 \equiv a(\text{mod} q)$, $p_1 \leq q$, $p_2 \leq q$ can be solved for all but $cq^{2\theta_q-1}\log^5 q$ residue classes $a \not\equiv 0$ modulo $q$;
(ii) if $H(\theta_q, q)$ is true with $\theta_q < 1-(3+\epsilon)\frac{\log\log q}{\log q}$ then $p_1p_2p_3 \equiv a(\text{mod} q)$, $p_1 \leq q$, $p_2 \leq q$, $p_3 \leq q$ can be solved;
(iii) if the generalized Riemann hypothesis is true then $p_1p_2 \equiv a$ (mod q), $p_1 \leq cq\log^4 q$, $p_2 \leq cq\log^4 q$ can be solved.

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11N13 Primes in progressions
11N05 Distribution of primes

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