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Erdős, Paul; Maier, H.; Sárközy, A.
On the distribution of the number of prime factors of sums $a+b$. (In English)
Trans. Am. Math. Soc. 302, 269-280 (1987). [0002-9947]

Let A and B be two subsets of positive integers not exceeding x. Let their cardinalities be denoted by $|A|$ and $|B|$. For any positive integer n put $\nu(n) = \sum_{p|n} 1$ and $\Omega(n) = \sum_{p^\alpha||n} \alpha$. For real v, put $\Phi(v) = (2\pi)^{-} \int_{-\infty}^{v} e^{-u^2/2} du$. The authors continue a series of investigations by A. Balog, P. Erdős and A. Sárközy. The result of this paper is a surprising link of $\nu(a+b)$ and $\Omega(a+b)$ ($a \in A$, $b \in B$) with the normal distribution function $\Phi(v)$ as in the famous Erdős-Kac theorem.

The authors prove the following Theorem: There exist absolute constants x_0, C_1 such that if $x > x_0$ and ℓ is any arbitrary positive integer then we have

$$||\{(a,b);\quad a \in A,\quad b \in B,\quad \nu(a+b) \leq \ell\}| \quad - \Phi(\frac{\ell - \log\log x}{(\log\log x)^{1/2}}) \quad |A| \quad |B||$$

$$< C_1 x(|A| \quad |B|)^{1/2} (\log\log x)^{-1/4}.$$

The same is true with $\Omega(a+b)$ in place of $\nu(a+b)$.

The theorem is proved by using the Hardy-Littlewood method. For $\ell = 0, 1, 2, \ldots$ put

$$S(x, \ell, \alpha) = \sum_{n \leq x, \nu(n) \leq \ell} e(n\alpha)$$

where $e(\alpha) = \exp(2\pi i \alpha)$. Also put

$$E(x, \ell) = \Phi(\frac{(\ell - \log\log x)}{(\log\log x)^{1/2}}).$$

The main lemma of the paper is the following: There exist absolute constants x_1, C_2 such that for $x > x_1$, $\ell = 0, 1, 2, \ldots.$ and any real number α, we have

$$|S(x, \ell, \alpha) - E(x, \ell) \sum_{n=1}^{[x]} e(n, \alpha)| \leq C_2 x (\log\log x)^{-1/4}.$$

The method of proof of this lemma bears some resemblance with that of Vinogradov applied in the proof of his three primes theorem.

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Classification:
11K65 Arithmetic functions (probabilistic number theory)
11P55 Appl. of the Hardy-Littlewood method
11P32 Additive questions involving primes

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arithmetic properties; dense sequences; sum sequences; Vinogradov method; normal distribution function; Erdős-Kac theorem; Hardy-Littlewood method