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Erdős, Paul; Sárközy, A.; Sós, V.T.

Problems and results on additive properties of general sequences. IV. (In English)

Number theory, Proc. 4th Matsci. Conf., Ootacamund/India 1984, Lect. Notes Math. 1122, 85-104 (1985).

[For the entire collection see Zbl 547.00014. - Part I, see the first and second author, Pac. J. Math. 118, 347-357 (1985; Zbl 569.10032).]

Let $\mathcal{A} = \{a_1 < a_2 < ...\}$ be an infinite sequence of positive integers and $R_1(n)$, $R_2(n)$, $R_3(n)$ denote the number of solutions of $a_x + a_y = n, \quad a_x \in \mathcal{A}, \quad a_y \in \mathcal{A}$ in the cases: no restriction, $x < y$, $x \leq y$, respectively. It turns out that these functions behave quite different according to monotonicity.

The authors show that $R_1(n)$ is monotonous increasing iff $\mathcal{A}$ consists of all the integers from a point onwards. Denoting the number of elements of $\mathcal{A}$ up to n by A(n) they construct sequences $\mathcal{A}$ such that $R_2(n)$ is monotonous increasing and $A(n) < n - cn^{1/3}$. There is no corresponding result for $R_3(n)$, however it is proved that $R_3(n)$ and $R_2(n)$ cannot be monotonous increasing when $A(n) = o(n/\log n)$. The authors conjecture that this is true with $A(n) = o(n)$.

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Classification:
11P99 Additive number theory
11B13 Additive bases
05B10 Difference sets
00A07 Problem books

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number of additive representations; infinite sequence of positive integers; monotonicity