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Erdős, Paul; Frankl, P.; Füredi, Z.
Families of finite sets in which no set is covered by the union of r others. (In English)
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If $\mathcal{F}$ is a collection of k-subsets of a set X, $X = \{1, 2, ..., n\}$, $\mathcal{F}$ is said to be r-cover free if $F_0 \subset / F_1 \cup F_2 \cup ... \cup F_r$, for every distinct $F_0, F_1, ..., F_r$. Denoting by $f_r(n, k)$ the maximum number of k subsets of X which satisfy the above condition, it is proved that $\binom{n}{t}/\binom{k}{t}^2 \leq f_r(n, k) \leq \binom{n}{t}/\binom{k-1}{t-1}$ for every n, k and r (where $t = [k/r]$) and that $f_r(n, r(t-1)+1+d) \leq \binom{n-d}{t}/\binom{k-d}{t}$ for $d = 0, 1$ or $d \leq r/2t^2$. Equality holds iff there exists a Steiner system $S(t, r(t-1)+1, n-d)$. Particular cases of r-cover free collections (which provide lower bounds for $f_r(n, tr)$) are the families introduced as near t-packing: a collection of tr-subsets of X ($t, r \geq 2$) is a near t-packing if $|F \cap F'| \leq t$, and $|F \cap F'| = t$ implies $\max\{i : i \in F\} \notin F'$ (for example, the collection $\{\{1, 2, 3, 5\}, \{1, 2, 4, 6\}, \{1, 3, 4, 7\}, \{2, 3, 4, 8\}\}$ is a near 2- packing in $\binom{8}{4}$. This is a generalization, in certain sense, of the concept of BIBD. This work is a continuation of a previous paper by the same authors, where they studied the problem of 2-cover free families of sets.

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05A99 Classical combinatorial problems

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