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Minimal decomposition of all graphs with equinumerous vertices and edges into mutually isomorphic subgraphs. (In English)

Finite and infinite sets, 6th Hung. Combin. Colloq., Eger/Hung. 1981, Vol. I, Colloq. Math. Soc. János Bolyai 37, 171-179 (1984).

[For the entire collection see Zbl 559.00001.]

Let $\mathcal{G} = \{G_1, G_2, ..., G_k\}$ be a set of graphs, all of the same size. A U-decomposition of $\mathcal{G}$ is a set of partitions of the edge sets E_i of the $G_i's$, $E_i = \cup_{j=1}^r E_{ij}$ such that for each fixed $j = 1, ..., r$, all the $E_{ij}(1 \le i \le k)$ induce isomorphic graphs. Denote by $U(\mathcal{G})$ the least value of r any U-decomposition of $\mathcal{G}$ can have, and by $U_k(n)$ the largest value of $U(\mathcal{G})$ over all sets $\mathcal{G}$ of k graphs of order n (and the same size).

It was shown by the authors, *S.M.Ulam*, and *F.F.Yao* [Congr. Numerantium 23, 3-18 (1979; Zbl 434.05046)] that $U_2(n) = 2/3n + o(n)$, and by the authors [Combinatorica 1, 13-24 (1981; Zbl 491.05049)] that $U_k(n) = 3/4n + o(n)$ for any fixed $k \ge 3$.

In the present paper, the family $\mathcal{G} = \mathcal{G}(n, e)$ of all graphs of order n and size e is investigated. Let $U(n)$ be the maximum value of $U(\mathcal{G}(n, e))$ over all values of e; clearly $U_k(n) \le U(n)$. The main result states that $U(n) = 3/4n + o(1)$; in particular, U$(\mathcal{G}(n, e)) = o(n)$ if $n/e = o(1)$.

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Classification:
05C35 Extremal problems (graph theory)

Keywords:
decomposition of edge sets of a collection of graphs; decomposition of graphs into mutually isomorphic subgraphs

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