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The asymptotic behavior of a family of sequences. (In English)

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A class of sequences defined by nonlinear recurrences involving the greatest integer function [.] is studied, a typical member of the class being $a(0) = 1$, $a(n) = a([n/2]) + a([n/3]) + a([n/6])$ for $n \geq 1$. For this sequence, it is shown that $\lim a(n)/n$ as $n \to \infty$ exists and equals $12/(log 432)$. More generally, for any sequence defined by $a(0) = 1$, $a(n) = \sum_{i=1}^{s} r_i a([n/m_i])$ for $n \geq 1$, where $r_i > 0$ and the m_i are integers ≥ 2, the asymptotic behavior of $a(n)$ is determined. Let τ be the unique solution to $\sum_{i=1}^{s} r_i m_i^{-\tau} = 1$. When there isan integer d and integers u_i such that $m_i = d^{u_i}$ for all i, $a(n)/n^\tau$ oscillates, while in the other case, where no such d and u_i exist, the limit of $a(n)/n^\tau$ exists and is explicitly computed. Results on the speed of convergence to the limit are also obtained.

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Classification:
11B37 Recurrences
11A25 Arithmetic functions, etc.
11B99 Sequences and sets

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nonlinear recurrences; greatest integer function; asymptotic behaviour; speed of convergence; limit; renewal theory; square functional equation