Zbl 541.10002

Erdős, Pál

On prime factors of binomial coefficients. II. (In Hungarian)

Mat. Lapok 30, 307-316 (1982). [0025-519X]

[For part I, cf. the author and *R. L. Graham*, Fibonacci Q. 14, 348- 352 (1976; Zbl 354.10010).]

This paper contains some results and unsolved problems concerning the prime factorization of the binomial coefficient $\binom{n}{k}$. Let $V(n,k)$ be the contribution of prime powers p^a to $\binom{n}{k}$ with $k<p\le n-k$. It is proved that if $m(n)$ denotes the greatest number satisfying $V(m(n),k)\le V(n,k)$ then $m(n)\gg n^{1+1/k}$. Let $f(k)$ resp. $F(k)$ be the smallest resp. greatest number satisfying $\omega\left(\binom{f(k)}{k}\right)\ge k$ resp. $\omega\left(\binom{F(k)}{k}\right)<k$. Is it true that $F(k)>f(k)$? Is it true that $\log f(k)=(1+o(1))e\log k$? It is proved that $F(k)\le A_k$, where A_k is the smallest common multiple of the integers up to k. Is it true that $F(k)<\exp((1-c)k)$?

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