Zbl 491.10044
Erdős, Paul
Some problems on additive number theory. (In English)
Ann. Discrete Math. 12, 113-116 (1982).

Let $f(n)$ be the largest integer k for which there is a sequence $1 \le a_1 < \cdots < a_k \le n$ such that all $a_i + a_j$ are distinct. The author and *P. Turán* have conjectured that $f(n) = n^{1/2} + O(1)$ and have proved that $n^{1/2} - n^{1/2-c} < f(n) < n^{1/2} + n^{1/4} + 1$ [J. Lond. Math. Soc. 16, 212- 215 (1941; Zbl 061.07301)]. Now let $m, n_1, \ldots, n_m, c$ be positive integers, let $A = \{A_1, \ldots, A_m\}$ be a system of sequences of integers $A_i = \{a_{i,1} < \cdots < a_{i,n_i}\}$, $i = 1, \ldots, m$, and let $D_i = \{a_{i,j} - a_{i,k} | 1 \le k < j \le n_i\}$ be the difference set of A_i. The system $S = \{D_1, \ldots, D_m\}$ is called perfect for c if the set $D = \cup_{i=1}^m D_i$ consists of the integers t such that $c \le t \le c - 1 + N$ where $N = \sum_{i=1}^n \binom{n_i}{2}$. J.Abrham has proved [Ann. Discrete Math. 12, 1-7 (1982)] that, for every perfect system, $m > \alpha N$ where $\alpha > 0$ is an absolute constant. The method that the author and Turán used to obtain their upper bound for $f(n)$ is now used to t if the integers $a_{i,j} - a_{i,k}$ in each D_i are all distinct and in $[1, N]$ and if $D_{i_1} \cap D_{i_2} = \emptyset$ for all $1 \le i_1 < i_2 \le m$, then for every $\varepsilon > 0$ there is an $\eta > 0$ so that, for $N > N_0(\varepsilon, \eta)$, if $|D| > (1+\varepsilon)N/2$ then $m > \eta N$. This theorem is then used to establish Abrham's result.

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05B10 Difference sets
11B75 Combinatorial number theory

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