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Erdős, Paul; Lint, J.H.van
On the average ratio of the smallest and largest prime divisor of n. (In English)
Indag. Math. 44, 127-132 (1982).

Let $p(n)$ and $P(n)$ denote respectively the smallest and the largest prime divisor of the integer $n, n \geq 2$. In 1974, J. van de Lune showed that $\sum_{n\leq x} \frac{p(n)}{P(n)} = o(x), x \to \infty$. In this paper the authors sharpen this to

$$\sum_{n\leq x} \frac{p(n)}{P(n)} = \frac{x}{\log x} + \frac{3x}{(\log x)^2}(1+o(1)), x \to \infty.$$

In a first section they prove the simpler estimate $\pi(x)(1+o(1))$ for the sum by fairly elemtary methods: splitting the positive integers n up tp x into three classes: (i) the primes and primes powers, (ii) the numbers with two different prime factors, (iii) the rest. These classes are in turn subdivided and of each subclass the contribution to the sum is estimated. The main term is of course due to the primes. In a second section the subdivisions and applied methods of estimating the varous sums are much more refined. The primes contribute, according to the prime number theorem $\frac{x}{\log x} + \frac{x}{\log x^2}(1+o(1))$. The integers of the form pq, p, q prime, $p \neq q$, give a contribution $(2x/(\log x)^2)(1+o(1))$ and all the rest goes into the remainder term. Although rather complicated, the proof is clearly presented and therfor easy to follow.

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Classification:
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11N37 Asymptotic results on arithmetic functions
11A41 Elemementary prime number theory

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