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**Erdős, Paul; Simonovits, M.**

*On the chromatic number of geometric graphs.* (In English)

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In this paper, serveral kinds of geometric graphs which can be associated to a metric space $U$ with metric $\rho$ are studied for the case $U \subseteq \mathbb{E}^h$, where $\mathbb{E}^h$ is the $h$-dimensional euclidean space. Let $G(U)$ be the graph with vertex set $U$ and which has as edge set the set of all pairs $x, y \in S$ with $\rho(x, y) = 1$, then the essential chromatic number of $\mathbb{E}^h$ is defined as $\chi_e(\mathbb{E}^h) = \{t|G(U)$ can be made $t$- chromatic by deleting $o(|S|^2)$ edges, and $U$ is finite$\}$. The following bounds for $\chi_e(\mathbb{E}^h)$ are given : 1. $\chi_e(\mathbb{E}^h) \leq 2$, 2. $\chi_e(\mathbb{E}^h) \geq h-2$ for $h \geq 2$, 3. $\chi(S^{h-1}) \leq \chi_e(\mathbb{E}^{2h}) \leq \chi_e(\mathbb{E}^{2h+1}) \leq \chi(S^h)$, where $\chi(S^{h-1})$ is the ordinary chromatic number of the sphere $S^{h-1}$ of radius $1/\sqrt{2}$ in $\mathbb{E}^h$. Furthermore, it is shown that for $h \geq 2$ the essential chromatic number of $\mathbb{E}^h$ coincides with the orthogonal chromatic number of $\mathbb{E}^h$ which is defined by: Given a set $\mathcal{P}$ of 2-dimensional subspaces of $\mathbb{E}^h$, let $\hat{G}(\mathcal{P})$ be the graph with vertex set $\mathcal{P}$ and which has as edge set the set of all pairs $P, Q \in \mathcal{P}$ with $P \perp Q$, then the orthogonal number is $\chi_\perp(\mathbb{E}^h) = \max\{\chi(G(\mathcal{P}))|\mathcal{P}$ is finite$\}$. Further results in this paper deal with embeddings of finite graphs in $\mathbb{E}^h$. The dimension of a finite graph $G$ is defined as $\dim(G) = \min\{h|G$ is contained in $G(U)$ as a subgraph, $U \subseteq \mathbb{E}^h$ is finite$\}$; the faithful dimension of $G$ is $\mathrm{Dim}(G) = \min\{h|G$ is isomorphic to $G(U)$, $U \subseteq \mathbb{E}^h$ is finite$\}$. These parameters are related to the maximum valence $\Delta(G)$ of $G$ by: 4. $\dim(G) \leq \Delta(G) + 2$, 5. $\mathrm{Dim}(G) \leq 2\Delta(G) + 1$.

*M. Walter*

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