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Values of the divisor function on short intervals. (In English)
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Let $\tau(n)$ denote the number of divisors of n. The following two theorems are proved. I. For every fixed positive integer k one has

$$\sum_{n<k} \max\{\tau(n), \tau(n+1), \dots \tau(n+k-1)\} \sim kx \log x, \qquad x \to \infty.$$

This result still holds when k depends on x, $\lim_{x\to\infty} k = \infty$, provided that $k = o((x \log x)^{3-2\sqrt{2}})$, $x \to \infty$. For the minimum taken by the divisor function on an interval of length k the problem turns out to be much more difficult. The sharpest result obtained here is the following. II. If k is a fixed positive integer and $\alpha_k = k(2^{1/k} - 1)$, then

$$\frac{c_1(k) x (\log x)^{\alpha_k}}{(\log\log x)^{11k^2}} \le \sum_{n<x} \min\{\tau(n), \tau(n+1), \dots \tau(n+k+1)\} \le C_2(k) x (\log x)^{\alpha_k}.$$

The proofs of the first theorem and the right hand inequality of the second theorem are elementary and presented in a series of six short lemmata. The left hand inequality of the second theorem is proved by an application of a lower bound form of the Selberg sieve.

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11A25 Arithmetic functions, etc.
11N35 Sieves

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