Zbl 431.10031
Erdős, Paul; Katai, I.
On the concentration of distribution of additive functions. (In English)
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The authors investigate the singularity or absolute continuity of distribution functions for certain classes of additive functions. A number- theoretic function f is said to be additive if $f(mn) = f(m) + f(n)$ whenever m and n are relatively prime positive integers. Also, f is strongly additive if $f(p^k) = (f(p))^k$ for all primes p. The function f has a limiting distribution if the frequencies $(1/n)\sharp\{n|f(n) \leq x\}$ converge weakly to a distribution function $F(x)$. Necessary and sufficient conditions for the existence and continuity of distribution functions of additive functions are classical by now, but it seems to be a difficult problem to distinguish between absolute continuity and singularity in particular cases. For an interesting discussion of these problems refer to the book by *P.D.T.A.Elliott* [Probabilistic number theory. I, II (1979 und 1980; Zbl 431.10029 und 431.10030)]. The main results of the paper are contained in four theorems. If F is distribution function, then $Q(h) = \sup_x(F(x + h) - F(x))$ is called the concentration function of F. The authors prove some results involving concentration functions of a class of strongly additive functions, thereby generalizing corresponding theorems for some particular additive functions. We state one of these results here. Theorem: Let $D(y) = \sum_{p<y} |f(p)|/p$, and suppose that $D(t^c) < 1/t$ and $|f(p_1) - f(p_2 9| > 1/t$ if $p_1 \neq p_2 < t^\delta$ hold, for suitable positive constants c and δ, for every large t. Then $(\log t)^{-1} \ll Q(\log t)^{-1}$ as $t \to \infty$. This theorem was proved for $f(n) = \log(\varphi(n)/n)$ by *M.M.Tjan* [Litov. Mat. Sb. 6, 105-119 (1966; Zbl 163.29201)] , and for $\log(\sigma(n)/n)$ by *P.Erdős* [Pac. J. Math. 52, 59-65 (1974; Zbl 291.10040)].

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