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Transversals and multitransversals. (In English)
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Let I be a set (of indices). Then, by definition, the disjoint subset relation $(a_i : i \in I) \to (b_i : i \in I)_{ds}$ means that the a_i, b_i are cardinals with the property that whenever A_i is a set of cardinality a_i there exist pairwise disjoint sets $X_i \in [A_i]^{b_i}$ $(i \in I)$. Families $(X_i : i \in I)$ are called multitransversals of $(A_i : i \in I)$ of size $(b_i : i \in I)$. A transversal [multitransversal] of a family F of sets is a family of distinct elements [disjoint sets] one from each number of F [cf. also §9, pp. 89-97 in reviewer's Thesis, Ensembles ordonnés et ramifiés, Paris (1935) and Publ. Math. Univ. Belgrade 4, 1-138 (1935; Zbl 014.39401)]. The authors consider 21 statements and prove the mutual equivalence of the statements (1), (2), (3), (4), (5) (Theorem 1), of statements (6), (7), (8), (9), (10) (Theorem 2), of statements (13), (14), (15), (16) (Theorem 3), respectively, and the following main results: "
Theorem 4: Let I be a set a_i, b_i be arbitrary cardinals for $i \in I$. Put $S = \{a_i : i \in I; b_i \geq i\}$. Then (17) $\leftrightarrow$ (18) $\leftrightarrow$ (19) $\wedge$ (20) $\wedge$ (21), where (17) $(a_i : i \in I) \to (b_i : i \in I)_{ds}$, (18) $(a_i : i \in I)$ has a multitransversal of size $(b_i : i \in I)$; (19) $\Sigma(i \in I; a_i \leq k)b_i \leq k$ for every cardinal k; (20) $\omega(S) \wedge \bar{\lambda} \notin \mathrm{stat}\lambda$ for every weakly inaccessible cardinal λ; (21) if $m < \omega$ and $m \leq \Sigma(i \in I; a_1 = \aleph_0)b_i$, then $m + \Sigma(i \in I;\, a_i \leq n)b_i \leq n$ for sufficiently large finite n. "Notation: For a cardinal c, c denotes the set of all cardinals $< c$. For a regular cardinal λ, a set A is stationary on $\bar{\lambda}$ if $A \subset \bar{\lambda}$ and for every regressive function f on A there exists $y < \lambda$ such that $|f^{-1}\{y\}| = \lambda$, $\mathrm{stat}\lambda$ denotes the system of all sets which are stationary on $\bar{\lambda}$.

:Kurepa

Classification:
04A20 Combinatorial set theory
04A25 Axiom of choice and equivalent propositions
04A10 Ordinal and cardinal numbers; generalizations
03E10 Ordinal and cardinal arithmetic

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transversal; stationary sets; disjoint subset relation; multitransversals; inaccessible cardinal