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Characterizing cliques in hypergraphs. (In English)

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The paper studies the set-theoretic analogue of a result of MacWilliams on affine spaces, with generalisations. Let $\binom{X}{r}$ denote the collection of all r-element subsets of X. A subset E of $\binom{X}{r}$ is an r-uniform hypergraph, and a subset of the form $\binom{Y}{r}$ for some $Y \subset X$ is a clique. If $|X| = n$, $|E| = 1$, the authors examine the question: for which n,l,r,j is it true that cliques are characterised by the property that, for any j-set S, $|E \cap \binom{S}{r}| = \binom{h}{r}$ for some h $(o \leq h \leq n)$? This holds if n is sufficiently large (depending on l, r and j); the authors find sharp bounds and characterise extremal cases. Stronger results are obtained for graphs $(r = 2)$.

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Classification:
04A20 Combinatorial set theory
05C65 Hypergraphs
05A05 Combinatorial choice problems
05C35 Extremal problems (graph theory)

Keywords:
combinatorial set theory; uniform hypergraph; subset; clique