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Deza, M.; Erdős, Paul; Frankl, P.

Intersection properties of systems of finite sets. (In English)

Combinatorics, Keszthely 1976, Colloq. Math. Janos Bolyai 18, 251-256 (1978).

[For the entire collection see Zbl 378.00007.]

Let X be a finite set of cardinality n. If $L = \{l_1, \ldots, l_r\}$ is a set of non-negative integers $l_1 < l_2 < \cdots < l_r$, and k is a natural number then by an (n, L, k)-system we mean a collection of k-element subsets of X such that the interesection of any two different sets has cardinality belonging to L. We prove that if $\mathcal{A}$ is an (n, L, k)-system, $|\mathcal{A}| > cn^{r-1}$ ($c = c(k)$ is a constant depending on k) then (i) there exists an l_1-element subset D of X such that D is contained in every member of $\mathcal{A}$, (ii) $(l_2 - l_1)|/l_3 - l_2)| \ldots |(l_r - l_{r-1})|(k - l_r)$, (iii) $\prod_{i=1}^{r} \frac{n-l_i}{k-l_i} \geq |\mathcal{A}|$ for $n \geq n_0(k)$).
Parts of the results are generalized for the following cases: (a) we consider t-wise intersections, $t \geq 2$, (b) the condition $|A| = k$ is replaced by $|A| \in K$ where K is a set of integers, (c) the intersection condition is replaced by the following: among $q + 1$ different members $A_1, \ldots, A_{q+1}$ there are always two subsets A_i, A_j such that $|A_i \cap A_j| \in L$. We consider some related problems. An open question: Let $L' \subset L$. Does there exist an (n, L, k)-system of maximal cardinality ($\mathcal{A}$) and an (n, L', k)-system of maximal cardinality ($\mathcal{A}$) such that $\mathcal{A} \supset \mathcal{A}'$?

Classification:
05A05 Combinatorial choice problems