Zbl 363.04009
Cates, M.; Erdős, Paul; Hindman, N.; Rothschild, B.
Partition theorems for subsets of vector spaces. (In English)
J. Comb. Theory, Ser. A 20, 279-291 (1976).

Partition theorems are considered also in vector spaces (cf. the papers of *P.Erdős* and *R.Rado* in Bull. Am. Math. Soc. 62, 427-489 (1956; Zbl 071.05101) and of *P.Erdős, A. Hajnal* and *R.Rado* in Acta Math. Acad. Sci. Hung. 16, 93-196 (1965; Zbl 158.26603)).
In particular, for any given quadruple $(\alpha, \beta, \gamma, \delta)$ of cardinals let $\langle\alpha\rangle \to \langle\beta\rangle^{\delta}_{\gamma}$ mean the following: Whenever V is an α-dimensional vector space over $GF(2)$ and $V = \bigcup_{\sigma<\gamma} A_\sigma$ there are some $U \in [V]^\beta$ and some $\sigma < \gamma$ such that if $1 \le t < \delta$ and $W \in [U]$ then $\Sigma W = A_\sigma$. In section 2 the authors prove 19 lemmas; e.g. L.r:3(a): If β, γ, $\delta < \omega$, then there exists the least integer $N(\beta, \gamma, \delta)$ such that $\langle N(\beta,\gamma,\delta)\rangle \to \langle\omega\rangle^{\omega}_{\gamma}$. L.r: 9: If β is a regular cardinal ¿ ω and $\langle\beta\rangle \to \langle\beta\rangle\_\gamma^3$ then $\beta \to (\beta)^2_\gamma$; L.2:10: GCH implies that every infinite nonlimit cardinal β satisfies $\langle\beta\rangle \nrightarrow \langle\beta\rangle^3_2$.
The main result (Th. 3:1): Assume the GCH and that there is no inaccessible cardinal $> \omega$. Exclude the possibility that any of the conditions: (a), (b) or (c) holds:

$$(a) \qquad \delta < \omega, \beta < \omega, \gamma = \aleph_\rho \qquad \text{and} \qquad \aleph_{\rho+\delta-1} \le \alpha < \aleph^{\beta-1}_{\rho+2};$$

$$(b) \qquad \delta = 4, \gamma < \omega, \beta = \aleph_\rho > \text{cf}(\beta) = \omega \qquad \text{and} \qquad \alpha < \aleph_{\rho+t(\gamma)}, t(\gamma) := 2\sum_{i=0}^{\gamma} \frac{r!}{i!} - 1;$$

$$(c) \qquad \delta = 4, \gamma < \omega, \text{cf}(\beta) > \omega, \beta = \aleph_\rho \qquad \text{and} \qquad \aleph\rho < \alpha < \aleph_{\rho+t(\gamma)}.$$

Then $\langle\alpha\rangle \to \langle\beta\rangle^\delta_\gamma$ holds if and only if one of the following 10 statements holds:
(1) $\gamma = 1$; (2) $\delta = 2$ and $\beta = 1$;
(3) $\delta = 2$, $\alpha \ge \omega$, $\gamma < \alpha$, and $\beta < \alpha$;
(4) $\delta = 2$, $\alpha \ge \omega$, $\gamma < \text{cf}(\alpha)$, and $\beta = \alpha$;
(5) $\delta = 3$, $\beta < \alpha$, $\alpha > \omega$, and $\gamma^+ < \alpha$;
(6) $\delta = 3$, $\beta = \alpha > \omega$, $\text{cf}(\alpha) = \omega$, and $\gamma < \omega$;
(7) $\delta = 4$, $\gamma < \omega$, $\beta = \aleph_\rho$, and $\alpha \ge \aleph_{\rho+t(\gamma)}$;
(8) $\beta < \omega$, $\gamma < \omega$, $\alpha \ge N(\beta, \gamma, \delta)$;
(9) $\beta < \omega$, $\gamma = \aleph_\rho$, and $\alpha \ge \aleph_{\rho+2^{\beta-1}}$;
(10) $\beta = \omega$ and $\gamma < \omega$.

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