Let n be a natural number and let $\Delta(n)$ denote any of the numbers for which the function $g_m(n) = m^{-1} \sum_{d|n, d \leq m} d$ assumes its maximum. It is proved that

$$\Delta(n) \geq \exp(c_1 \log n / \log \log n)$$

for all n large enough,

$$\Delta(n) \leq \exp(c_2 \log n / \log \log n)$$

for infinitely many n's with suitble positive constants c_1, c_2 and $\Delta(n) < n$ for almost all n.

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Classification:
11K65 Arithmetic functions (probabilistic number theory)
11N05 Distribution of primes