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*Subgraphs with all colours in a line-coloured graph.* (In English)

**Proc. 5th Br. comb. Conf., Aberdeen 1975, 101-112 (1976).**

[For the entire collection see Zbl 316.00006.]

Consider a graph $G$ whose lines have been coloured. It is natural to look for conditions on the colouring, which ensure that a particular kind or class of coloured graphs exist as subgraphs of $G$. This paper deals with one such problem. Let $p, q$ and $r$ be positive integers with $r \geq 3$ and $2r \geq q \geq r$. The function $g(p, r, q)$ [resp. $f(p, r, q)$] is defined as the least integer $\ell \geq 0$ such that, whenever a graph [resp. complete grpah] on $p$ points has each of its lines coloured with one of $r-1$ colours [resp. $r$ colours] in such a way that every colour is on strictly more than $\ell$ lines, then the graph has a polychromatic subgraph on $\leq q$ points (polychromatic means containing all colours). Any $\ell \geq \binom{p}{2}/r$ variously satisfies the condition for $f$, however, the authors prefer not to take such $\ell - s$ into consideration, and they define $f$ to be $\infty$ when only such values of $\ell$ exist. In the paper a variety of results about $g$ and $f$ are proved, among them: $g(p, 3, 3) = \binom{[p/2]}{2}$ for $p \geq 1$, $f(p, r, 2r-2) = g(p, r, 2r-2) = 0$ for $r \geq 3$, and $f(p, r, 2r-3) = g(p, r, 2r-3) = \binom{\alpha}{2}$ for $r \geq 4$ and $\alpha = [p/(r-1)]$. Also some asymptotic results are obtained. Finally it is proved (for $r \geq 3$ and $p > (1/3)r + 2$) that if the lines of the complete $p$-graph are coloured with $r$ colours, each colour being used at least once, then there is a polychromatic trail of length $\leq 2r-3$ (a trail is a linesequence in which all lines are distinct). This is stronger than $f(p, r, 2r-2) = 0$.

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Classification:
05C15 Chromatic theory of graphs and maps
05C35 Extremal problems (graph theory)