
Zbl 337.10005
Erdős, Paul
On asymptotic properties of aliquot sequences. (In English)
Math. Comput. 30, 641-645 (1976). [0025-5718]

If n is a positive integer the aliquot sequence $\{s^i(n)\}$ with leader n is defined as follows: $s^o(n) = n$ and $s^{k+1}(n) = \sigma(s^k(n)) - s^k(n)$ for $k \geq 0$. The Catalan-Dickson conjecture states that every aliquot sequence is bounded (so that either $s^k(n) = 1$ for some k or the sequence becomes periodic). Guy and Selfridge, however, are "tempted to conjecture" that the Catalan-Dickson conjecture is false. The main result of the present paper is as follows: for every positive integer k and every positive real number δ

$$(*) \qquad (1-\delta)n(s(n)/n)^i < s^i(n) < (1+\delta)n(s(n)/n)^i, \quad 1 \leq i \leq k$$

for all n except a sequence of density zero. Since $s(n)/n \geq 7/5$ for $n \equiv 0($ mod 30), (*) implies that for every k there exists an m such that $s^o(m) < s(m) < s^2(m) < \ldots < s^k(m)$. The result just stated was first proved by H. W. Lenstra, and his proof is published for the first time in the present paper.

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Classification:
11B37 Recurrences
11A25 Arithmetic functions, etc.