
Zbl 329.10036
Erdős, Paul; Hall, R.R.
Distinct values of Euler's φ-function. (In English)
Mathematika, London 23, 1-3 (1976).

Let $V(x)$ denote the number of distinct values not exceeding x taken by Euler's φ-function, so that $\pi(x) \le V(x) \le x$. In a previous paper by the authors [Acta arithmetica 22, 201-206 (1973; Zbl 252.10007)], they show that for each fixed $B > 2\sqrt{(2/\log 2)}$, the estimate $V(x) \ll \pi(x)\exp\{B\sqrt{(\log\log x)}\}$ holds. In this paper they show that there exist positive absolute constants A, C, such that

$$V(x) \geq C \quad \pi(x)\exp\{A(\log\log\log x)^2\}.$$

The methods used involve the number of representations of n in the form $n = m_i(p-1)$ where p is a prime and the sequence of distinct numbers of the form $(p_1-1)(p_2-1)\dots(p_k-1)$ subject to various conditions. The authors conclude by asking the question: is it true that, for every $c > 1$, $\lim V(cx)/V(x) = c$?

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Classification:
11N37 Asymptotic results on arithmetic functions
11A25 Arithmetic functions, etc.