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On the existence of triangulated spheres in 3-graphs, and related problems. (In English)

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The problem described in the title represents an analogue of the well known property of graphs that any graph on n vertices and having at least n edges contains a polygon. That result could be restated, in topological terms, as saying that any simplicial 1-complex with at least as many 1-simplexes as 0-simplexes must contain a triangulation of the 1-sphere. In Theorem 3 we shall determine asymptotically the maximum number of 2-simplexes a simplicial 2-complex may contain without containing a subcomplex which is a triangulation of the 2-sphere. More precisely, we shall prove that there exist constants c_1 and c_2 such that every 3-graph on n vertices having $c_2 n^{3/2}$ edges or more contains a double pyramid; but that there exists a 3-graph on n vertices having $c_1 n^{3/2}$ edges containing no triangulation of the sphere. Also, we discuss several related results.

Classification:

05C10 Topological graph theory

57M20 Two-dimensional complexes

05C35 Extremal problems (graph theory)