Zbl 222.05007
Erdős, Paul; Schönheim, J.
On the set of non pairwise coprime divisors of a number. (In English)
Combinat. Theory Appl., Colloquia Math. Soc. János Bolyai 4, 369-376 (1970).

[For the entire collection see Zbl 205.00201.]
Theorem 1. If $D_1, \dots, D_m$ are different divisors of an integer N whose decomposition in prime factors is $\prod_{i=1}^t p_i^{\alpha_i}$ and each two of the d's have a common divisor > 1 then, denoting $\prod_{i=1}^t \alpha_i = \alpha$,

$$\max m = f(N) = \frac{1}{2} \sum \max \left\{ \prod_{\nu=1}^{\mu} \alpha_{i_\nu}, \alpha / \prod_{\nu=1}^{\mu} \alpha_{i_\nu} \right\}$$

where the summation is over all subsets $\{i_1, \dots, i_\mu\}$ of $\{1, \dots, t\}$ and for the empty subset the product is considered to be one. The result is best possible, for every N there are $f(N)$ divisors no two of which are relatively prime. Theorem 2. Let $G_1, \dots, G_m$ be m distinct divisors of N not two of which are relatively prime. Assume $m < g(N)$. Then there are $g(N) - m$ further divisors $G_{m+1}, \dots, G_{g(N)}$ so that no two of the $g(N)$ distinct divisors G_i, $1 \le i \le g(N)$ are relatively prime. Theorem 2 is best possible.

Classification:
05A17 Partitions of integres (combinatorics)