
Zbl 219.10064**Erdős, Paul***Remarks on number theory. III: On addition chains* (In English)**Acta Arith. 6, 77-81 (1960). [0065-1036]**

[Part II, Acta arithmetica 5, 171-177 (1959; Zbl 092.04601)]

An addition chain is a sequence $1 = a_0 < a_1 < \dots < a_k = n$ of integers such that every a_l ($l \geq 1$) can be written as the sum $a_i + a_j$ of two preceding members of the sequence. Define $l(n)$ to be the smallest k for which such a sequence exists. *A. Brauer* [Bull. Am. Math. Soc. 45, 736-739 (1939; Zbl 022.11106)] has shown that $\lim_{n \rightarrow \infty} l(n) \log 2 / \log n = 1$ and that, for all n ,

$$(1) \quad l(n) < \frac{\log n}{\log 2} + \frac{\log n}{\log \log n} + O\left(\frac{\log n}{\log \log n}\right).$$

The present author now shows that (1) holds with equality for almost all n . The methods of proof are typically Erdős. The generalisation to the case where each a_l can be written as the sum of at most r (≥ 2) preceding members of the sequence is briefly dealt with, and similar results are stated.

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Classification:

11B75 Combinatorial number theory

11B83 Special sequences of integers and polynomials