Zbl 201.33704
Erdős, Paul
On a combinatorial problem. II (In English)
Acta Math. Acad. Sci. Hung. 15, 445-447 (1964). [0001-5954]

A family F of subsets of a set M is said to have property B if there exists a subset K of M so that no set of the family F is contained either in K or $\bar{K}$ (the complement of K in M). *P. Erdős* and *A. Hajnal* [ibid. 12, 87-123 (1961; Zbl 201.32801)] investigated property B and its generalizations, and posed the problem: What is the smallest integer $m(n)$ for which there exists a family F, of sets $A_1, A_2, \ldots, A_{m(n)}$ each having n elements, which does not possess property B? They observed that $m(n) \leq \binom{2n-1}{n}$, $m(1) = 1$, $m(2) = 3$, $m(3) = 7$. The author[Nordisk mat. Tidskr. 11, 5-10 (1963; Zbl 116.01104)] showed that $m(n) > 2^{n-1}$ for all n and that for $n > n_0(\epsilon)$, $m(n) > (1-\epsilon)2^n \log 2$. *W. M. Schmidt* [Acta. Math. Acad. Sci. Hung. 15, 373-374 (1964; Zbl 143.02501)] proved $m(n) > 2^n/(1+4/n)$. *H. L. Abbott* and *L. Moser* [Can. Math. Bull. 7, 177-181 (1964; Zbl 131.01302)], using a constructive method, proved $m(ab \leq m(a)(m(b))^a$ and from this deduced that for $n > m_0$, $m(n) < (\sqrt{7}+\epsilon)^n$ and that $\lim_{n\to\infty} m(n)^{1/n}$ exists. In the present paper the author uses a non- constructive method to show that $m(n) < n^2 2^{n+1}$ (and hence $\lim_{n\to\infty} m(n)^{1/n} = 2$), and suggests that $m(n) = 0(n2^n)$ is a reasonable guess.

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Classification:
05D05 Extremal set theory
04A20 Combinatorial set theory