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Erdős, Paul; Hajnal, András
On a property of families of sets (In English)
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A family $\mathcal{F}$ of sets is said to have property B if there exists a set S meeting all but containing none but one of the members of $\mathcal{F}$, i.e. $0 < |S \cap F| < |F|$ for every $F \in \mathcal{F}$. The study of property B was started by *E.W.Miller* [C. R. Soc. Sci. Varsovie 30, 31- 38 (1937; Zbl 017.30003)]. The authors systematically investigate conditions on a family of sets in order that it does or does not have property B or a related property. Important examples of the author's results are the following.
(1) Let $\mathcal{F}$ be a family of at most $\aleph_\omega$ sets, each of power $\aleph_1$ and with the intersection of any two finite. Then $\mathcal{F}$ has property B. (Whether the result remains true for more than $\aleph_\omega$ sets is unsettled..)
(2) Let $\mathcal{F}$ have $\aleph_k$ infinite sets, k finite, any pair of which intersect in at most one element. Then there exists a set S meeting each in at least one and at most $k+2$ elements, i.e. $|S \cap F| \in [1, k+2]$ for all $F \in \mathcal{F}$, and $k+2$ is best possible.
Many unsolved problems are stated, some of which have since been partially solved in the literature. Soon to be published is "Unsolved problems in set theory" (Axiomatic Set Theory, Proc. Symp. Pure and Applied Math., A.M.S.) in which the same authors summarize the current state of research on these and related problems.

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Classification:
05D05 Extremal set theory
04A20 Combinatorial set theory