Zbl 186.37902

Erdős, Pál

On the boundedness and unboundedness of polynomials (In English)

J. Anal. Math. 19, 135-148 (1967). [0021-7670]

Let $x_i^{(j)}$, $1 \leq i \leq j$ be numbers in the closed intervall $[-1, 1]$ strictly increasing with i for each fixed j. For each n let P_n denote a polynomial of degree n in x. The author proves a necessary and sufficient condition on the triangular matrix $(x_i^{(j)})$ that the following implication hold. If for each $m, n(1 + c) < m$, and for each $i, 1 \leq i \leq m$, we have $|P_n(x_i^{(n)})| \leq 1$, then there exists a function $A(c)$ depending only on c such that $\max(|P_n(x)| : -1 \leq x \leq 1)$ is less than $A(c)$.
The proof is difficult, and is related with earlier work of the same author [cf. the author, Ann. of Math., II. Ser. 44, 330-337 (1943; Zbl 063.01266)]. The result proved extends results of Zygmund and Berstein concerning the Tchebycheff and Legendre polynomials respectively.

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Classification:
26C05 Polynomials: analytic properties (real variables)
33C25 Orthogonal polynomials and functions