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**Zbl 186.07903****Erdős, Pál; Szemerédi, E.***On a problem of P. Erdős and S. Stein* (In English)**Acta Arith. 15, 85-90 (1968). [0065-1036]**

The system of congruences (1)  $a_i \pmod{n_i}$ ,  $n_1 < \dots < n_k$ , is called a covering system if every integer satisfies at least one of the congruences (1). An old conjecture of Erdős states that for every integer  $c$  there is a covering system with  $n_1 = c$ . This is still unproved. A system (1) is called disjoint if every integer satisfies at most one of the congruences (1). Denote by  $f(x)$  the largest value of  $k$  for which there is a disjoint system (1) satisfying  $n_k \leq x$ . Erdős and Stein conjectured that  $f(x) = o(x)$ . The authors prove a stronger result, namely Theorem 1 below. Theorem 1. For every  $\varepsilon > 0$  if  $x > x_0(\varepsilon)$  we have ( $c_1$  denotes a suitable positive constant)

$$x / \exp((\log x)^{1/2+\varepsilon}) < f(x) < x / (\log x)^{c_1}.$$

*P. Chowla*

Classification:

11A07 Congruences, etc.

11B25 Arithmetic progressions