Zbl 177.52502

Erdős, Pál

On the number of complete subgraphs and circuits contained in graphs (In English)

Čas. Pěstování Mat. 94, 290-296 (1969).

Let $G(n;k)$ be a graph of n vertices and k edges. K_p denotes a complete graph of p vertices. Let $n \equiv r(\mod p-1)$, $m(n;p = \frac{p-2}{2(p-1)}(n^2-r^2)+\binom{r}{2}$, $0 \le r \le p{-}1$. A well known theorem of Turán states that every $G(n;m(n;p){+}1)$ contains a K_p and that this result is best possible. Denote by $f_n(p;1)$ the largest integer so that every $G(n;m(n;p)+1)$ contains at least $f_n(p;1)$ $K_p's$. The author proves that for $n > n_0(p)$

$$f_n(p;1) = \prod_{i=0}^{p-1} \left[\frac{n+1}{p-1}\right].$$

In particular $f_{3n}(4,1) = n^2$. Several further results are proved, $f_n(p,1)$ is determined for $1 < \varepsilon_p n$ and several unsolved problems are stated. [See also P. Erdős, Illinois J. Math. 6, 122-127 (1962; Zbl 099.39401)]

Classification:
05C20 Directed graphs (digraphs)
05C38 Paths and cycles