Zbl 173.03903
Erdős, Pál
Some remarks on the iterates of the φ and σ functions (In English)
Colloq. Math. 17, 195-202 (1967). [0010-1354]

Put $\varphi_1 = \varphi(n)$, $\varphi k(n) = \varphi_1(\varphi_{k-1}(n))$ and denote by $N_\varphi(k, \alpha, x)$ the number of integers $n \leq x$ for which $\varphi_k(n) > \alpha n$. The author proves that for every $t, \alpha < \frac{1}{2}$, $\varepsilon > 0$, and $x > x_0(\alpha, t, \varepsilon)$ the inequalities hold

$$x(\log\log x)^t / \log x < N_\varphi(2, \alpha, x) < x(\log x)^\varepsilon / \log x;$$

further, for every $\alpha > 0, \varepsilon > 0$ and $x < x_0(a, \varepsilon)$

$$N_\varphi(3, \alpha, x) < x(\log x)^\varepsilon / (\log x)^2.$$

A similar theorem for the σ function is given without proof and many related question are discussed.

A.Schinzel

Classification:
11A25 Arithmetic functions, etc.
11N64 Characterization of arithmetic functions