Zbl 154.29403
Erdős, Pál
Remarks on number theory. I (In Hungarian)
Mat. Lapok 12, 10-16, 161-168 (1961). [0025-519X]

I. Denote by $n_k(p)$ the smallest positive k-th power non-residue (mod p). Mirsky asked the author to find an asymptotic formula for $\sum_{p\le x} n_k(p)$. The author proves using the large sieve of Linnik if $p_1 < p_2 < \cdots$ is the sequence of consecutive primes that

$$\sum_{p\le x} n_2(p) = (1+o(1)) \sum_{k=1}^{\infty} \frac{p_k}{2^k} \frac{x}{\log x}.$$

It is very likely true that $\sum_{p<x} n_k(p) = (1+o(1)) \frac{c_k x}{\log x}$.
II. Let $\varphi(n) = \varphi_1(n)$ be Euler's φ function and put $\varphi_k(n) = \varphi(\varphi_{k-1}(n))$. The author proves that if we neglect a sequence of density 0 then for $k \ge 2$

$$\lim_{n\to\infty} \varphi_k(n) \frac{\log\log\log n}{\varphi_{k-1}(n)} = c^{-c},$$

where C is Euler's constant. Several other problems and results are stated about the φ function.

Classification:
11N69 Distribution of integers in special residue classes
11A25 Arithmetic functions, etc.