
Zbl 151.33204**Erdős, Pál; Harary, Frank; Tutte, W.T.***On the dimension of a graph* (In English)**Mathematika, London 12, 118-122 (1965).**

The following concept of the dimension of a graph is presented: The dimension G of a graph G is the minimum number n such that G can be embedded into Euclidean n -space E_n with every edge of G having length 1. Some of the results: For the complete graph K_m with m vertices $\dim K_m = m - 1$; for the complete bicoloured graph $K_{m,n}$, $\dim K_{m,n} \leq 4$, for n -dimensional cube Q_n , $n > 1$, is $\dim Q_n = 2$ for all n . For any graph G with chromatic number $\chi(G)$, $\dim G \leq 2 \cdot \chi(G)$.

If $\dim G = 2$, then $\chi(G) \leq 7$. Among all graphs with n vertices, q edges, and dimension $2k$ or $2k + 1$, $\lim_{n \rightarrow \infty} \max qn^{-2} = \frac{1}{2}(1 - k^{-1})$. Among any n points of E_4 the distance 1 between pairs of points can occur at most $n + \lfloor n^2/4 \rfloor$ times, and this number can be realized if $n \equiv 0 \pmod{8}$.

E. Jucovič

Classification:

05C35 Extremal problems (graph theory)

05C15 Chromatic theory of graphs and maps

05C10 Topological graph theory