
Zbl 131.21001
Erdős, Pál; Rényi, Alfréd
On a problem in the theory of graphs (In Hungarian)
Publ. Math. Inst. Hung. Acad. Sci., Ser. B 7, 623-639 (1963).

Let $H_2(n,k)$ denote the set of all (non-directed) graphs G_n having n prescribed vertices, in which the maximum of the valencies of the vertices is equal to k, and the diameter of which is ≤ 2. We put $F_2(n,k) = \min N(G_n)$, $G_n \in H_2(n,k)$ where $N(G)$ denotes the number of edges of the graph G. [If $H_2(n,k)$ is empty we put $F_2(n,k) = +\infty$.] The following inequalities are proved: Theorem 1. $F_2(n,k) \geq n(n-1)/2k$. Theorem 2. $F_2(n,k) \geq n(n-1)/(k+8n/k)$ if $k^2 > 8n$. It is shown further by effective construction that Theorem 1 is asymptotically best possible, and that Theorem 2 is also asymptotically best possible in the case $k^2/n \to +\infty$. The constructions are based on the use of finite geometries.

Classification:
05C35 Extremal problems (graph theory)