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Let $a_1 < a_2 < \cdots$, $A(x) = \sum_{a_i \le x} 1$ be an infinite sequence for which (1) $a_k = a_{i_1} + \cdots + a_{i_r}$, $i < \cdots < i_r < k$, is not solvable. I prove that $A(x)/x \to 0$ and that $\sum \frac{1}{a_i} < 103$. Further I show that $A(x) = o(x)$ is best possible, but there always exists a sequence $x_i \to \infty$ for which (2) $A(x) < Cx_i^{(\sqrt{5}-1)/2}$. On the other hand, there exists a sequence A for which (1) has no solutions, but $A(x) > cx^{2/7}$ for every x. Perhaps (2) can be improved, but the exponent can certainly not be made smaller than 2/7. Consider now the sequences A for which the equation (1') $a_{r_1} + \cdots + a_{r_{s_1}} = a_{l_1} + \cdots + a_{l_{s2}}$, $r_1 < \cdots < r_{s_1}$; $l_1 < \cdots < l_{s_2}$; $s_1 \ne s_2$, is not solvable for every choice of $s_1 \ne s_2$. There exists such a sequence with $A(x) > c_x^\alpha$ for every x if α is sufficiently small. On the other hand, I show by using Rényi's strengthening of the large sieve of Linnik that if A is such that (1') has no solutions, then $A(x) < cx^{5/6}$ for every x if c is a sufficiently large absolute constant. Perhaps the exponent 5/6 can be improved, but I have not succeeded in doing this.

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