Zbl 109.16502
Erdős, Pál
On circuits and subgraphs of chromatic graphs (In English)
Mathematika, London 9, 170-175 (1962).

A graph is k-colorable if its points can be colored using k colors in such a way that no two points of the same color are adjacent. The chromatic number of a graph is k if it is k-colorable but not $(k-1)$-colorable. Let $g_k(n)$ be the largest integer for which there is a graph of n points of chromatic number k and girth (minimum cycle length) $g_k(n)$. Let $f(m,k;n)$ be the maximum chromatic number among all graphs of n points, every subgraph of which is k-colorable. Theorem I: For $k \geq 4$, $g_k(n) \leq 1 + 2\log n/\log(k-2)$. Theorem 2: To every k there is an $\varepsilon > 0$ so that if $n > n_0(\varepsilon, k)$ there exists a graph with n points and chromatic number k every subgraph of which having $[\varepsilon n]$ points is 3- colorable. Theorem 3: For $m > 3$ $f(m,3;n) > c(n/m)^{1/3}[\log(n/2m)^{-1}]$.

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05C15 Chromatic theory of graphs and maps