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Erdős, Pál; Rényi, Alfréd
On the evolution of random graphs (In English)
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A random graph $\Gamma_{n,N}$ is a undirected finite graph without parallel edges and slings. $\Gamma_{n,N}$ has n points $P_1, ..., P_n$ and N edges (P_i, P_j), which are chosen at random so that all $\binom{\binom{n}{2}}{N} = C_{n,N}$ possible choices are supposed to be equiprobable. Let be $P_{n,N}(A) = A_{n,N}/C_{n,N}$ the probability that $\Gamma_{n,N}$ has the property A, where $A_{n,N}$ denotes the number of graphs with the given points $P_1, ..., P_n$, with N edges (P_i, P_j) and with the property A. $\Gamma_{n,N}$ is studied under the condition that N is increased, i.e. if N is equal, or asymptotically equal, to a given function $N(n)$ of n. For many properties A there is shown that there exists a "threshold function" $A(n)$ of the property A tendig monotonically to $+\infty$ for $n \to +\infty$ such that $\lim_{n\to+\infty} P_{n,N(n)}(A) = 0$ or $=1$ if $\lim_{n\to+\infty} \frac{N(n)}{A(n)} = 0$ or $= +\infty$. $A(n)$ is a "regular threshold function" of A if there exists a probability distribution function $F(x)$ such that $\lim_{n\to+\infty} P_{n,N(n)}(A) = F(x)$ if $\lim_{n\to+\infty} \frac{N(n)}{A(n)} = x$, where $0 < x < +\infty$ and x is a point of continuity of $F(x)$. The investigated properties are as follows: the presence of certain subgraphs (e. g. trees, complete subgraphs, cycles, etc.) or connectedness, number of components etc. The results are of the following type: Theorem 3a. Suppose that $N(n) \sim cn$, where $c > 0$. Let γ_k denote the number of cycles of order k contained in $\Gamma_{n,N}$ $(k = 3, 4, ...)$. Then we have $\lim_{n\to+\infty} P_{n,N(n)}(\gamma_k = j) = \lambda^j e^{-\lambda}/j!$, where $j = 0, 1, ...$ and $\lambda = (2c)^k/2k$. Thus the threshold distribution corresponding to the threshold function $A(n) = n$ for the property that the graph contains a cycle of order k is $1 - e^{-(2c)^k/2k}$.

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Classification:
05C80 Random graphs