Zbl 100.27104
Erdős, Pál
On a problem of S.W.Golomb. (In English)
J. Aust. Math. Soc. 2, 1-8 (1961).

A set of primes is defined in the following way: $q_1 = 3$, $q_2 = 5$, $q_3 = 17, ..., q_k$ is the smallest prime greater than q_{k-1} for which $q_k \not\equiv 1 \pmod{q_i}$ $1 \le i < k$. Let $A(x)$ denote the number of $q_i \le x$. *S.W.Golomb* (Zbl 067.27503) proved that $\underline{\lim}_{x\to\infty} \frac{A(x)\log x}{x} = 0$.
In this paper the author proves that $A(x) = (1 + o(1))\frac{x}{\log x \log\log x}$. The proof is based on use of Brun's method and results on primes in short arithmetic progression.
In the end the author states that by similar arguments the following more general result can be proved: Let $r > 1$, $Q_1 > r + 1$, Q_1 prime, and Q_{i+1} the smallest prime greater than Q_i such that $Q_{i+1} \not\equiv t \pmod{Q_j}$, $1 \le j \le i$, $1 \le t \le r$. Let further $B_{Q_1,r}(x)$ be the number of Q' not execceding x, then

$$B_{Q_1,r}(x) = (1 + o(1))x/\log x \log_2 x \cdots \log_{r+1} x$$

where $\log_k x$ denotes the k time iterated logarithm.
There are several misprints in the paper.

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Classification:
11N56 Rate of growth of arithmetic functions