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Erdős, Pál; Fodor, G.
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[Part V see Zbl 072.04103]
The paper is dealing with relations in a given set E of cardinality $m > \aleph_0$. For every $x \in E$ let $R(x)$ be a subset of E. Two distinct elements x, y of E are independent, if both $x \in R(y)$ und $y \in R(x)$. Every monopunctual subset of E as well as every subset of pairwise independent points is called a free subset of R. The following 3 conditions are considered:
(A) There exists a cardinality $n < m$ satisfying $kR(x) < n$ for every $x \in E$ (kX means the cardinality of X).
(B) E is a metric space and $\operatorname{dist}(x, R(x)) > 0$ for every $x \in E$.
(C) There exists a real number $r > 0$ such that, putting $g(x) = \operatorname{dist}(x, R(x))$, the set $\{x \mid g(x) \geq r\}$ contains in B a subset of positive measure; B denotes the system of all Borel sets of E; E is a metric space containing an everywhere dense set of a cardinality $< i$ (i denotes the first inaccessible cardinal number $> \aleph_0$). For a system S of sets a subsystem I of S is called a p-additive ideal provided (I) the union of every subsystem of I of cardinality $< p$ belongs to I and (II) for every $X \in I$ the relations $Y \subseteq X$, $Y \in S$ imply $Y \in I$.
Theorem 1. If $m = \aleph_\gamma > i$ and if I denotes a proper $\aleph_{\lambda+1}$-ideal of subsets of E such that $\{x\} \in I$ for every $x \in E$; if $B \subseteq E$, $B \notin I$, then there exists a disjointed $\omega_{\lambda+1}$-sequence B_ξ of subsets of E such that $B_\xi \notin I$ ($\xi < \omega_{\lambda+1}$) and $B = \bigcup B_\xi$.
This theorem is used in providing the following one (theorem 3): Under the conditions of Th. 1. if $R(x)$ is finite for every $x \in E$, then for every ω-sequence of subsets E_ξ of E such that $E_\xi \notin I$, there exists a free subset E' of E satisfying $E' \cap E_\xi \notin I$ for every $\xi < \omega$. Now suppose that the condition (B) holds. Let E denote the set of all real numbers and $kR(x) < \aleph_0$ ($x \in E$). Then there exists a freesubset E' of E such that E' be everywhere of the second category and that the Lebesgue outer measure $\mu(E')$ of E' be $b - a$ in every interval (a, b) (Th. 6). Let not E be an interval of real numbers and B be a σ-algebra of subsets of E containing all subintervals of E; let μ be a non-trivial measure on B. Then the condition (C) implies the existence in B of a free subset of positive μ-measure.
In the particular case when $R(x)$ is the complement of an interval of E whose center is at x, the converse holds too: the existence in B of a free subset of E of positive μ-measure implies the condition (C) (Th. 7). Theorem 11: Let K be a disjointed class of cardinality g of subsets of E of cardinality $m = kE$ each; then the condition (A) implies the existence of a free subset E' of E such that the cardinality of $X \cap E'$ be m for every $X \in K$ (here $m \geq \aleph_0$).

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Classification: