Zbl 071.05105
Erdős, Pál; Rado, R.
A partition calculus in set theory. (In English)
Bull. Am. Math. Soc. 62, 427-489 (1956).

The memoir is a natural sequel of some previous articles (Zbl 038.15301; Zbl 048.28203; Zbl 051.04003; Zbl 055.04903) initiated by *F.P.Ramsey* in 1930 [Proc. London math. Soc., II. Ser. 30, 264-286 (1930)], see also *D.Kurepa*, C. R. Soc. Sci. Varsovie, Cl. III 1939, 61-67; Acad. Sci. Slovenica, Ser. A 1953; Dissertationes IV/4, 67-92 (1953). For a set S and a cardinal r let $[S]^r = \{X| X \subseteq S, |X| = r\}$; in particular $[S]^r = 0$, provided $|S| < r$. The basic concept is the following relation: Given numbers a, k, r and a k-sequence b_ν $(\nu < k)$; the relation $a \to [b_0, b_1, ..., b_\nu, ...]^r_k$ is said to hold provided for a set S of cardinality a and for every partition of the set $[S]^r$: $S^r = \bigcup_{r<k} K_\nu$ there are a $B \subseteq S$ and a $\nu < k$ satisfying $|B| = b_\nu$, $[B]^r \subseteq K$. An analog relation is defined if a, b_ν be order types; in this case instead of $|B| = b$ one considers the condition $\bar{B} = b$ ($\bar{B}$ denoting the order type of B). If b_ν is a constant sequence b_0 the corresponding relation is denoted $a \to (b_0)^r_k$. The paper contains 50 theorems and several problems; some known theorems are included for the completion sake. Frequently the index k is dropped too; e.g. if Φ is an order type such that $\Phi \leq \lambda$, $|\Phi| > \aleph_0$ and if $\alpha < \omega_0 2$, $\beta < \omega_0^2$, $\gamma < \omega_1$, then $\Phi \to (\omega_0\gamma)^2$, $\Phi \to (\alpha, \beta)^2$ (Th. 5, and Zbl 048.28203, Theorems 5 and 7). The main problem is this: Is the relation $\lambda \to (\omega_0 2, \omega_0^2)^2$ true or false?

One of the main results reads (Th. 43): If $r < s \leq b_0$, $b_1 \to (s)^r_k$ then $\alpha \to (b_0, b_1)^2$ (this relation holds for order types as well as for cardinal numbers). If φ is an order type $> \aleph_0$ such that $\omega_1, \omega_1^* \not\leq \varphi$ and if $\alpha < \omega 2$, $\beta < \omega^2$, $\gamma < \omega_1$ then $\varphi \to (\alpha, \alpha, \alpha)^2 \wedge (\alpha, \beta)^2 \wedge (\omega, \gamma)^2 \wedge (4, \alpha)^3$ (Th. 31). Let $\alpha \to (\beta, \gamma)^2$; let m be the initial ordinal of cardinality $|\alpha|$; then $\beta < \omega_0 \vee \gamma < \omega_0 \vee \beta, \gamma \leq \alpha$, $m \vee \beta, \gamma \leq \alpha, m^*$ (Th. 19). If $\alpha < \omega_4$ then $\alpha \not\to (3, \omega 2)^2$, $\omega 4 \to (3, \omega 2)^2$ (Th. 24). If $r \geq 3$, then $\lambda \not\to (\omega, \omega + 2)^r$ (Th. 27). $|\lambda| \not\to (\aleph_1 \aleph_1)^r$ for $r \geq 2$ (Th. 30). For given r, k and β_ν $(\nu < k)$, there exists an ordinal α such that $\alpha \to (\beta_0, \beta_1, ..., \beta_\nu, ...)^r_k$ (Cor. Th. 39). Moreover canonical partition relation as well as polarized partition relations are considered (§§8, 9).

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Classification:
05D10 Ramsey theory
03E05 Combinatorial set theory (logic)
04A20 Combinatorial set theory