Zbl 019.23602

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Über Euler-Linien unendlicher Graphen.

On Eulerian lines in infinite Graphs. (In German)

J. Math. Phys., Mass. Inst. Techn. 17, 59-75 (1938).

König (Theorie der Graphen p. 31) posed the problem: When does a denumerably infinite graph G contain an Euler line (a chain Z extending infinitely in both directions and containing each edge of G exactly once)? The authors obtain these necessary and sufficient conditions: (T_1) G is connected; (T_2), G contains no vertex of odd order; (E_1) If g is any finite subgraph of G, $G-g$ has at most two infinite components; (E_2) If all vertices of a finite g have in g the same, even, order, then $G-g$ has only one infinite component. Necessary and sufficient that G contain an Euler line infinite in one direction are the conditions: (T_1); (T^*), G contains a vertex of either infinite or odd order, and at most one vertex of odd order; (E) each $G-g$ with g finite has at most one infinite component. The proof that these sets of conditions are sufficient depends in each case on removing a finite chain z containing a specified edge, adding to z all finite components C_i of $G-z$, applying the known finite methods to $g' = z + \sum C_i$, and finally showing that the remainting portions of G can be suitably attached to the Euler line of g' in virtue of the essential conditions (E). For this argument it suffices to assume weakened forms of (E_i) in which the finite g is only a chain or circuit containing a fixed vertex.

Applications: the existence of an Euler line for the lattice of n-space, conditions for the existence of a finite number of lines covering G; and the theorem that G has a Z containing each edge exactly twice if and only if (T_1) and (E) hold.

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Classification:
05C99 Graph theory