Zbl 012.01201
Erdős, Pál; Turán, Pál
Ein zahlentheoretischer Satz.
A number-theoretical theorem. (In German)
Mitteil. Forsch.-Inst. Math. Mech. Univ. Tomsk 1, 101-103 (1935).

Let a be a fixed integer, and let $l(k)$ be defined (for any k prime to a) as the least positive integer for which $a^{l(k)\equiv}1(\bmod k)$. Generalising a result of *N.P.Romanoff* (Zbl 009.00801), the authors prove here that $\sum_k \frac{1}{kl(k)^\varepsilon}$ converges for every $\varepsilon > 0$. It suffices to prove that $\sum \frac{1}{k}$ extended over those k for which $l(k) < (\log k)^{\frac{2}{\varepsilon}}$ converges. For this it suffices that the number of divisors $\leq n$ of $(a-1)(a^2-1)...(a^N-1)$ should be $O(n/\log^2 n)$, where $N = \left[(\log n)^{\frac{2}{\varepsilon}}\right]$. This is proved by estimating the number of prime factors, and considering separately those divisors with more than $\sqrt{\log n}$ different prime factors and those with less.

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Classification:
11B25 Arithmetic progressions
11N13 Primes in progressions